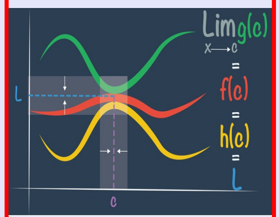
Calculus I

Lecture 27



Feb 19-8:47 AM

Class QZ 26

1) Find $\int \cos x \sin(\sin x)\,dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int \sin u\,du = -\cos u + C = \boxed{-\cos(\sin x) + C}$

2) Evaluate $\int_0^2 2x(x^2+1)^2\,dx$

$u = x^2 + 1$

$du = 2x\,dx$

$x = 0 \to u = 1$

$x = 2 \to u = 5$

$= \int_1^5 u^2\,du = \frac{u^3}{3}\Big|_1^5 = \frac{1}{3}\left[5^3 - 1^3\right] = \frac{1}{3}\cdot 124 = \boxed{\frac{124}{3}}$

May 28-8:44 AM

$f(x) = \int_1^{\sqrt{x}} \frac{t^2}{t^4+1}\,dt$

Find $f(1) = \int_1^{\sqrt{1}} \frac{t^2}{t^4+1}\,dt = \int_1^{1} \frac{t^2}{t^4+1}\,dt = \boxed{0}$

Find $f'(x) = \frac{(\sqrt{x})^2}{(\sqrt{x})^4+1} \cdot \frac{d}{dx}[\sqrt{x}] - \frac{1^2}{1^4+1} \cdot \cancel{\frac{d}{dx}[1]}^{\,0}$

$= \frac{x}{x^2+1} \cdot \frac{1}{2\sqrt{x}} - 0 = \frac{\sqrt{x}}{2(x^2+1)}$

$f'(x) > 0 \rightarrow f(x)$ is increasing.

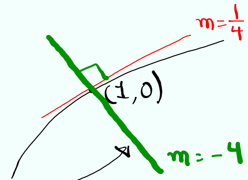
Find $f'(1) = \frac{\sqrt{1}}{2(1^2+1)} = \frac{1}{4}$

Find eqn of the normal line to the graph of $f(x)$ at $x=1$.

$y - y_1 = m(x - x_1)$

$y - 0 = -4(x-1)$

$\boxed{y = -4x+4}$



May 28-9:08 AM

Find $f'(x)$ for $f(x) = \int_{x^2}^{x^3} \sin^2 t^3\,dt$

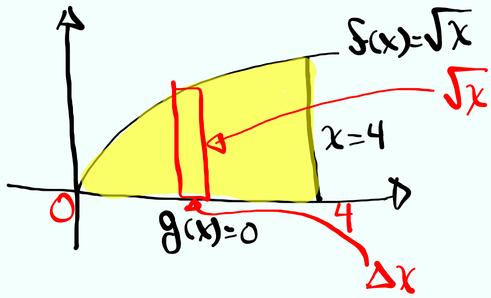
$f'(x) = \sin^2(x^3)^3 \cdot \frac{d}{dx}[x^3] - \sin^2(x^2)^3 \cdot \frac{d}{dx}[x^2]$

$= \sin^2 x^9 \cdot 3x^2 - \sin^2 x^6 \cdot 2x$

$= \boxed{3x^2 \sin^2 x^9 - 2x \sin^2 x^6}$

May 28-9:17 AM

Find the area between $f(x)=\sqrt{x}$, $g(x)=0$, and $x=4$. Drawing required.



$$A=\int_0^4 \sqrt{x}\,dx$$

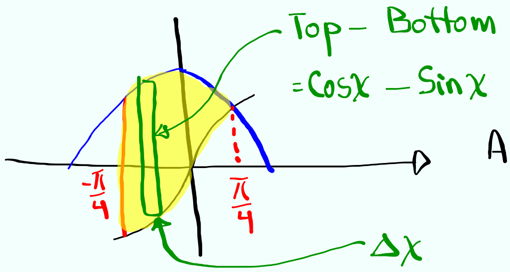
$$=\frac{x^{3/2}}{3/2}\Big|_0^4$$

$$=\frac{2}{3}x\sqrt{x}\Big|_0^4=\frac{2}{3}\left[4\sqrt{4}-0\sqrt{0}\right]$$

$$=\frac{2}{3}\cdot 8=\frac{16}{3}$$

May 28-9:22 AM

Find the area bounded by $f(x)=\sin x$, $g(x)=\cos x$ for $-\frac{\pi}{4}\le x\le\frac{\pi}{4}$. Drawing required.



$$A=\int_{-\pi/4}^{\pi/4}\left[\cos x-\sin x\right]dx$$

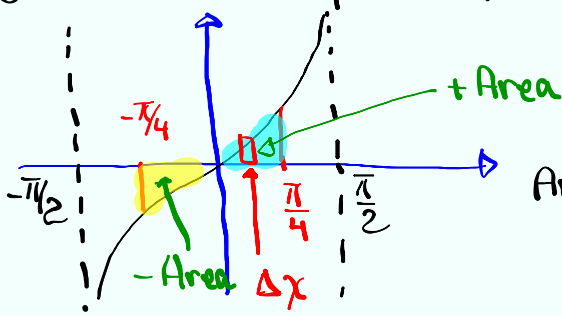
$$=(\sin x+\cos x)\Big|_{-\pi/4}^{\pi/4}=\left(\sin\frac{\pi}{4}+\cos\frac{\pi}{4}\right)-\left(\sin\frac{-\pi}{4}+\cos\frac{-\pi}{4}\right)$$

$$=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}-\left(\cancel{\frac{-\sqrt{2}}{2}}\cancel{+\frac{\sqrt{2}}{2}}\right)^{0}$$

$$\boxed{=\sqrt{2}}$$

May 28-9:29 AM

Find the area bounded by $f(x) = \tan x$, $g(x) = 0$ for $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$. Drawing Required.



$$\text{Area} = 2\int_0^{\pi/4} (\tan x - 0)\,dx$$

$$= 2\int_0^{\pi/4} \tan x\,dx$$

Calc. 2 $\longrightarrow$ $= 2\ln|\cos x|\Big|_0^{\pi/4}$

May 28-9:36 AM

$$f(x) = \int_{\sqrt{x}}^{x^3} \cos t^2\,dt \qquad f(1) = \int_{\sqrt{1}}^{1^3} \cos t^2\,dt = 0$$

$$f'(x) = \cos(x^3)^2 \cdot 3x^2 - \cos(\sqrt{x})^2 \cdot \frac{1}{2\sqrt{x}}$$

$$f'(1) = \cos 1 \cdot 3 - \cos 1 \cdot \frac{1}{2} = \cos 1\left(3 - \frac{1}{2}\right) = \boxed{\frac{5}{2}\cos 1}$$

$$f(x) = \int_0^x (1 - t^2)\cos^2 t\,dt$$

For what values of x $f(x)$ is increasing.

$$f'(x) = (1 - x^2)\cos^2 x \cdot 1 - (1 - 0^2)\cdot\cos^2 0 \cdot 0$$

$$f'(x) = (1 - x^2)(\cos^2 x) \to \ge 0$$

$1 - x^2 > 0$

$x^2 - 1 < 0$

$x = 1 \quad x = -1$

x		-1		1	
$f'(x)$	$-$		$+$		$-$

$\boxed{-1 < x < 1}$ $(-1, 1)$

$f'(x) > 0$

$f(x)$ increasing

May 28-9:58 AM

Volume

we take a region, rotate it about x-axis OR y-axis, we generate Volume → find that Volume.

3 - Methods

1) Disk

2) Washer

3) Shell

May 28-10:07 AM

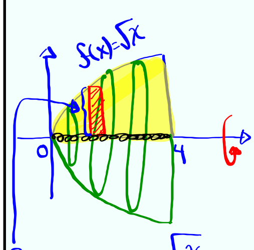
Disk Method

1) Region is totally attached to axis of revolution

2) Reference rectangle ⊥ to axis of revolution

$$V = \int_a^b \pi R^2 \, dx$$

R = height of ref. rectangle



1) Ref. Rect. ⊥ A.O.R. ✓

2) Region is totally attached to A.O.R. ✓

Radius = $R = \sqrt{x}$

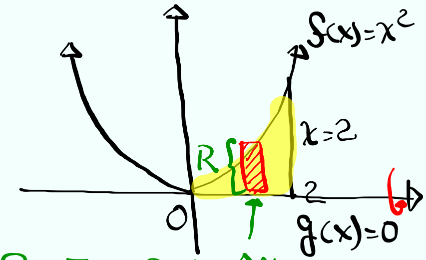
Disk Method

$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx = \pi \int_0^4 x \, dx = \pi \cdot \frac{x^2}{2} \Big|_0^4$$

$$= \frac{\pi}{2}\left[4^2 - 0^2\right] = \boxed{8\pi}$$

May 28-10:10 AM

Rotate the region bounded by $f(x)=x^2$, $g(x)=0$, $x=2$ by x-axis. Find the volume.



1) Ref. Rect. $\perp$ A.O.R. ✓

2) Region attached 100% to A.O.R. ✓

$R = \text{Top} - \text{Bottom}$ Δx

$= x^2 - 0$

$= x^2$

Disk Method

$$V = \int_0^2 \pi (x^2)^2 \, dx$$

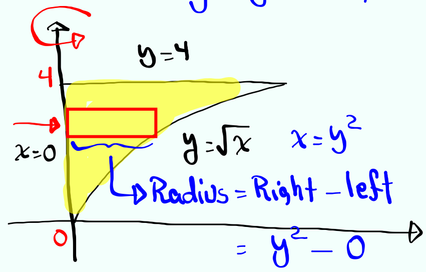
$$= \pi \cdot \frac{x^5}{5}\Big|_0^2 = \frac{\pi}{5}(2^5 - 0^5)$$

$$= \boxed{\frac{32\pi}{5}}$$

May 28-10:18 AM

Draw the region bounded by $y=\sqrt{x}$, $y=4$, and $x=0$.

Δy



$= y^2 - 0$

$= y^2$

Rotate by y-axis, find its volume.

1) Ref. Rect. $\perp$ A.O.R. ✓

2) Region 100% attached to A.O.R. ✓

Disk

$$V = \int_0^4 \pi [y^2]^2 \, dy = \pi \cdot \frac{y^5}{5}\Big|_0^4$$

$$= \frac{\pi}{5}(4^5 - 0^5) = \boxed{\frac{1024\pi}{5}}$$

May 28-10:24 AM

Draw the region bounded by $y=5-x^2$, and $y=1$.

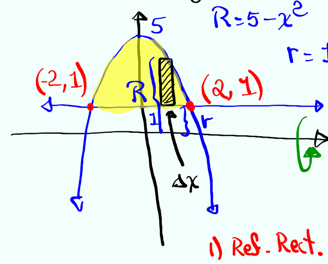
$5-x^2=1$

$-x^2=1-5$

$-x^2=-4$

$x^2=4$

$x=\pm 2$



1) Ref. Rect. ⊥ A.O.R.? Yes

Rotate by x-axis, find the Volume.

2) Is the region 100% attached to A.O.R.? NO

Washer Method

$$V=\int_a^b \pi(R^2-r^2)\,dx$$

$$V=\int_{-2}^{2}\pi\left[(5-x^2)^2-1^2\right]dx$$

$$=\pi\int_{-2}^{2}(25-10x+x^2-1)\,dx$$

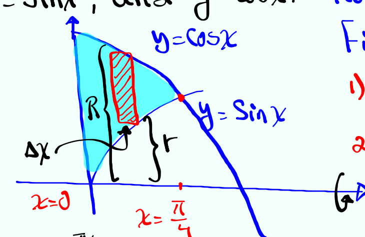
$$=\pi\int_{-2}^{2}(24-10x+x^4)\,dx$$

$$=\pi\cdot 2\left[24x-\frac{10x^2}{2}+\frac{x^3}{3}\right]\Big|_0^2=2\pi\left[48-20+\frac{8}{3}-0\right]$$

$$=2\pi\left[28+\frac{8}{3}\right]=\frac{184\pi}{3}$$

May 28-10:34 AM

Draw the region bounded by $x=0$, $x=\frac{\pi}{4}$, $y=\sin x$, and $y=\cos x$. Rotate by x-axis. Find the Volume.



1) Disk No

2) Washer Yes

$R=\cos x$

$r=\sin x$

$$V=\int_0^{\pi/4}\pi\left[R^2-r^2\right]dx=\pi\int_0^{\pi/4}\left[\cos^2 x-\sin^2 x\right]dx$$

$$=\pi\int_0^{\pi/4}\cos 2x\,dx$$

$u=2x$ $\quad$ $x=0\to u=0$

$du=2dx$ $\quad$ $x=\frac{\pi}{4}\to u=\frac{\pi}{2}$

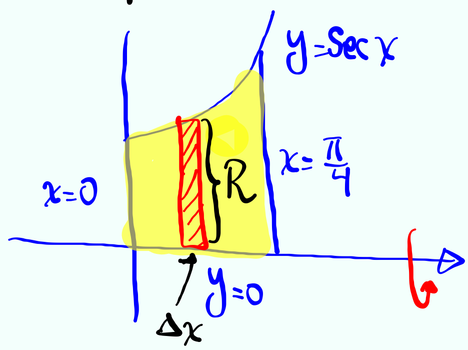
$\frac{du}{2}=dx$

$$=\pi\int_0^{\pi/2}\cos u\,\frac{du}{2}$$

$$=\frac{\pi}{2}\cdot\sin u\Big|_0^{\pi/2}=\frac{\pi}{2}\left[\overbrace{\sin\frac{\pi}{2}}^{1}-\overbrace{\sin 0}^{0}\right]=\boxed{\frac{\pi}{2}}$$

May 28-10:48 AM

Rotate the region bounded by $y = \sec x$, $y = 0$, $x = 0$, and $x = \frac{\pi}{4}$ by x-axis. Find its Volume.



Disk ✓

$R = \sec x$

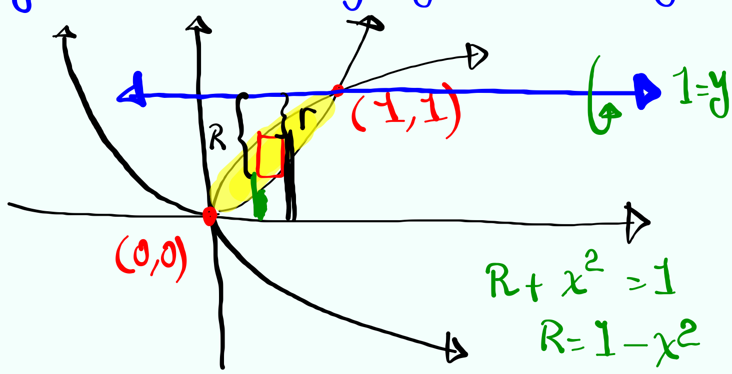
$$V = \int_0^{\pi/4} \pi [\sec x]^2 \, dx$$

$$= \pi \int_0^{\pi/4} \sec^2 x \, dx = \pi \tan x \Big|_0^{\pi/4} = \boxed{\pi}$$

May 28-10:58 AM

$\sqrt{x} = y$

Rotate the region bounded by $y = x^2$ & $x = y^2$ by $y = 1$.



washer

$R = 1 - x^2$

$r = 1 - \sqrt{x}$

$R + x^2 = 1$

$R = 1 - x^2$

$r + \sqrt{x} = 1$

$r = 1 - \sqrt{x}$

$$V = \int_0^1 \pi \left[(1 - x^2)^2 - (1 - \sqrt{x})^2 \right] dx$$

May 28-11:05 AM